EE 435

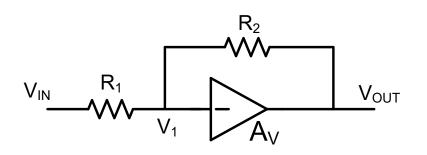
Lecture 3

Design Space Exploration

with applications to single-stage amplifier design

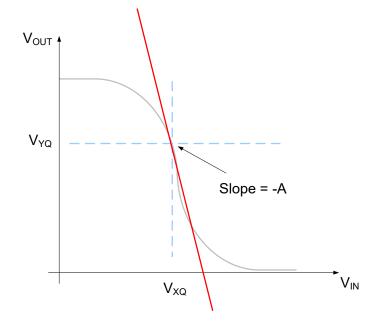
Systematic Strategies for Designing and Analyzing Op Amps

Single-ended Op Amp Inverting Amplifier



$$V_{O} = (-A)(V_{1}-V_{XQ})+V_{YQ}$$

$$V_{1} = \frac{R_{1}}{R_{1}+R_{2}}V_{O}+\frac{R_{2}}{R_{1}+R_{2}}V_{IN}$$

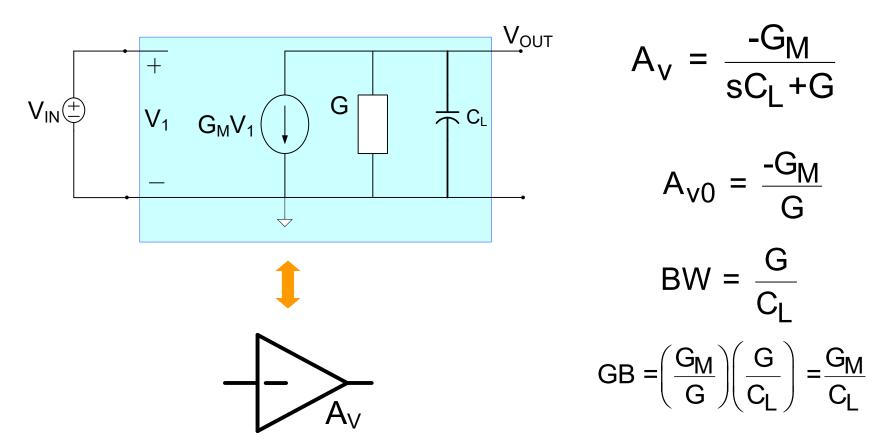


Summary:

$$V_{O} = -\frac{R_{2}}{R_{1}}V_{iss} + V_{XQ} + \frac{R_{2}}{R_{1}}(V_{XQ} - V_{inQ})$$

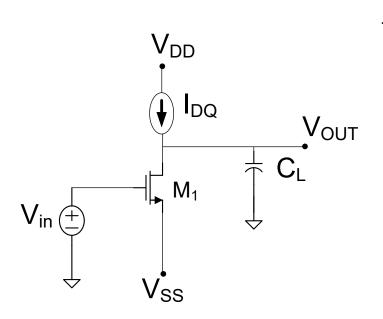
What type of circuits have the transfer characteristic shown?

Single-stage single-input low-gain op amp (unilateral with infinite input impedance and one capacitor)



GB and A_{VO} are two of the most important parameters in an op amp

How do we design an amplifier with a given architecture in general or this architecture in particular?



What is the design space?

Generally V_{SS} , V_{DD} , C_L and possibly V_{OUTQ}) will be fixed

Must determine $\{ W_1, L_1, I_{DQ} \text{ and } V_{INQ} \}$

Thus there are 4 design variables

But W₁ and L₁ appear as a ratio in almost all performance characteristics of interest

and I_{DQ} is related to V_{INQ} , W_1 and L_1 (this is a constraint)

Thus the design space generally has only two independent variables or two degrees of freedom $\left\{\frac{W_{_{1}},I_{_{DO}}}{\cdot}\right\}$

Thus design or "synthesis" with this architecture involves exploring the two-dimensional design space $\left\{ \underline{W}_{1,l_{DO}} \right\}$ 6

Review from last lecture: How do we design an amplifier with a given architecture?

- 1. Determine the design space
- 2. Identify the constraints
- 3. Determine the entire set of unknown variables and the Degrees of Freedom
- 4. Determine an appropriate parameter domain
- 5. Explore the resultant design space with the identified number of Degrees of Freedom

Parameter Domains for Characterizing **Amplifier Performance**

Degrees of Freedom: 2

Small signal parameter domain:

$$\{g_m,g_0\}$$

$$A_{v0} = \frac{-g_m}{g_0}$$

$$GB = \frac{g_m}{C_l}$$

Natural design parameter domain:

$$\left\{\frac{W}{L}, I_{DQ}\right\}$$

$$A_{V0} = \left[\frac{\sqrt{2\mu C_{OX}}}{\lambda}\right] \left[\frac{\sqrt{\frac{W}{L}}}{\sqrt{I_{DQ}}}\right] \qquad GB = \left[\frac{\sqrt{2\mu C_{OX}}}{C_{L}}\right] \left[\sqrt{\frac{W}{L}}\sqrt{I_{DQ}}\right]$$

$$GB = \left[\frac{\sqrt{2\mu C_{OX}}}{C_{L}}\right] \left[\sqrt{\frac{W}{L}}\sqrt{I_{DQ}}\right]$$

Alternate parameter domain:

$$\left\{ \mathsf{P,V}_{\mathsf{EB}}\right\}$$

$$A_{V0} = \left[\frac{2}{\lambda}\right] \frac{1}{V_{EB}}$$

$$GB = \left[\frac{2}{V_{DD}C_{L}}\right] \left[\frac{P}{V_{EB}}\right]$$

Parameter Domains for Characterizing **Amplifier Performance**

Degrees of Freedom: 2

Small signal parameter domain:

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$$A_{v0} = \frac{-g_m}{g_0}$$

$$GB = \frac{g_{m}}{C_{l}}$$

Natural design parameter domain:

$$\left\{\frac{W}{L}, I_{DQ}\right\}$$

$$A_{VO} = \left[\frac{\sqrt{2\mu \, C_{OX}}}{\lambda} \right] \frac{\sqrt{\frac{W}{L}}}{\sqrt{I_{DQ}}}$$

$$A_{VO} = \left[\frac{\sqrt{2\mu C_{OX}}}{\lambda} \right] \frac{\sqrt{\frac{W}{L}}}{\sqrt{I_{DQ}}} \qquad GB = \left[\frac{\sqrt{2\mu C_{OX}}}{C_{L}} \right] \sqrt{\frac{W}{L}} \sqrt{I_{DQ}}$$

Alternate parameter domain:

$$c_{VO} = \left[\frac{2}{\lambda}\right] \left[\frac{1}{V_{EB}}\right]$$

$$GB = \left[\frac{2}{V_{DD}C_L}\right] \left[\frac{P}{V_{EB}}\right]$$

$$\{P, V_{EB}\}$$

Parameter Domains for Characterizing **Amplifier Performance**

Degrees of Freedom: 2

Small signal parameter domain:

$$\{g_m,g_0\}$$

$$A_{v0} = \frac{-g_m}{g_0}$$

$$GB = \frac{g_{m}}{C_{l}}$$

Natural design parameter domain:

$$\left\{ \frac{\mathsf{W}}{\mathsf{L}},\mathsf{I}_{\mathsf{DQ}} \right\}$$

$$A_{VO} = \left[\frac{\sqrt{2\mu \, C_{OX}}}{\lambda}\right] \frac{\sqrt{\frac{W}{L}}}{\sqrt{I_{DQ}}} \qquad GB = \left[\frac{\sqrt{2\mu C_{OX}}}{C_{L}}\right] \sqrt{\frac{W}{L}} \sqrt{I_{DQ}}$$

Alternate parameter domain:

$$\{P,V_{EB}\}$$

$$A_{V0} = \left[\frac{2}{\lambda}\right] \left[\frac{1}{V_{EB}}\right] \qquad GB = \left[\frac{2}{V_{DD}C_L}\right] \left[\frac{P}{V_{EB}}\right]$$

$$GB = \left[\frac{2}{V_{DD}C_{L}}\right] \left[\frac{P}{V_{EB}}\right]$$

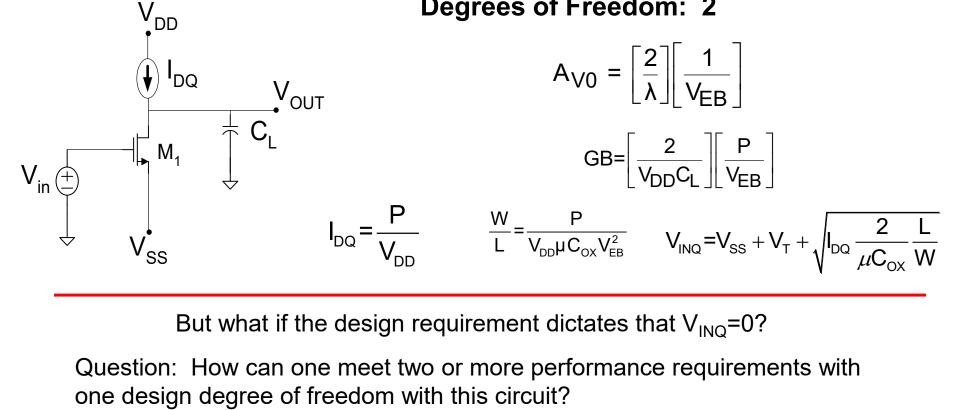
- Alternate parameter domain gives considerable insight into design
- Easy to map from alternate parameter domain to natural parameter domain
- Alternate parameter domain provides modest parameter decoupling
- $A_{V0} = \frac{\lambda}{2}$ and $A_{GB} = \frac{V_{DD}C_L}{2}$ figures of merit for comparing architectures

Design With the Basic Amplifier Structure

Consider basic op amp structure

Alternate parameter domain: $\{P, V_{EB}\}$

Degrees of Freedom: 2



one design degree of freedom with this circuit?

Degrees of Freedom: 1

Luck or Can't

Key Observation about Analog Circuit Design

Matching of R's, C's and MOSFETs and BJTs can be very good

.1%, .01%, or even better

Good layout is critical for good matching

Large area often need for good ratio accuracy

Good matching can be maintained over wide temperature range

Absolute value of R's, C's, and MOS and BJT parameters very bad

+/- 30% or worse

Termed Process/Voltage/Temperature (PVT) effects

Values can be highly temperature dependent

Several % variation over 100°C range or worse

Trimming/Calibration at production test can compensate parameter variations

Adds some cost for test

Trim elements consume considerable area

Temperature dependence still a challenge

How do we design an amplifier with a given architecture?

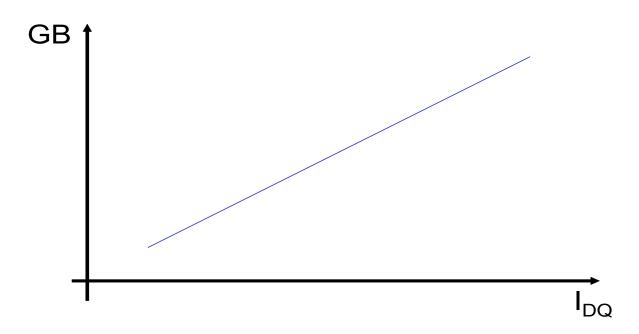
- 1. Determine the design space
- 2. Identify the constraints
- 3. Determine the entire set of unknown variables and the Degrees of Freedom
- 4. Determine an appropriate parameter domain



Question: How does the GB of the single-stage amplifier common-source amplifier change with bias current?

$$GB = \frac{g_{m}}{C_{L}} \qquad \longrightarrow \qquad GB = \left[\frac{2}{C_{L}}\right] \left[\frac{I_{DQ}}{V_{EB}}\right]$$

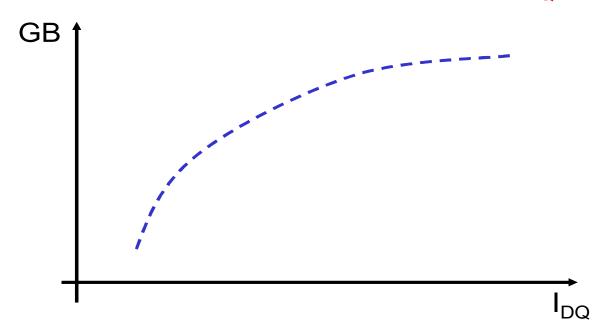
GB increases linearly with I_{DQ}



Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \left[\frac{\sqrt{2\mu C_{OX}}}{C_{L}}\right] \left[\sqrt{\frac{W}{L}}\sqrt{I_{DQ}}\right]$$

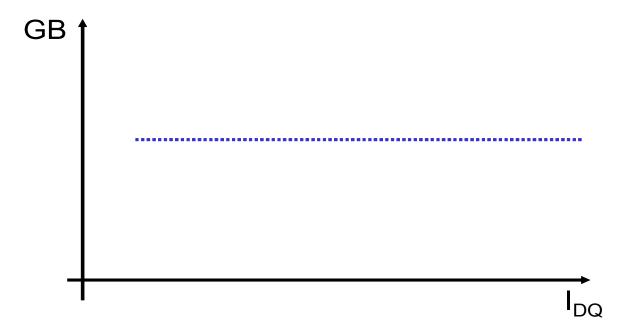
GB increases with the square root of I_{DQ}



Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \left[\frac{2}{V_{DD}C_{L}}\right] \left[\frac{P}{V_{EB}}\right]$$

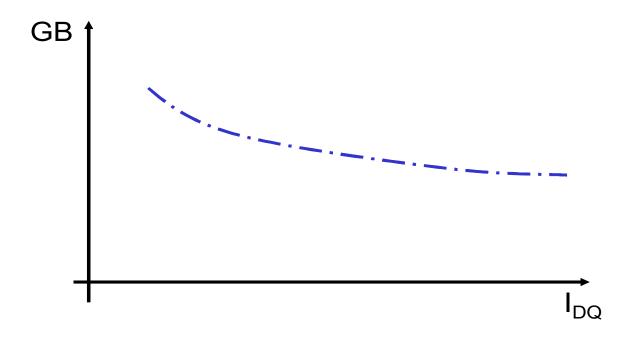
GB independent of I_{DQ}



Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \frac{1}{\sqrt{I_{DQ}}} \frac{P}{C_L} \sqrt{\frac{2\mu C_{OX}W}{L}}$$

GB decreases with the reciprocal of the square root of I_{DQ}

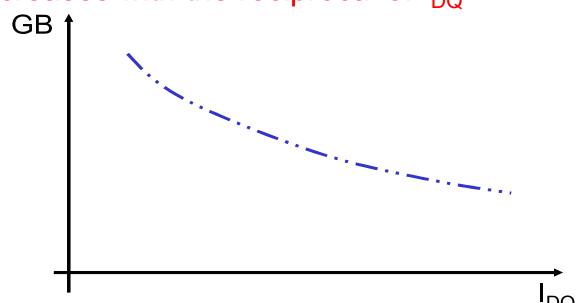


17

Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \frac{\sqrt{\frac{2\mu C_{OX}WP^3}{LV_{DD}}}}{I_{DQ}C_L}$$

GB decreases with the reciprocal of I_{DO}



Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \begin{bmatrix} \frac{2}{C_L} \end{bmatrix} \begin{bmatrix} I_{DQ} \\ V_{EB} \end{bmatrix} \qquad \qquad \text{Increases Linearly}$$

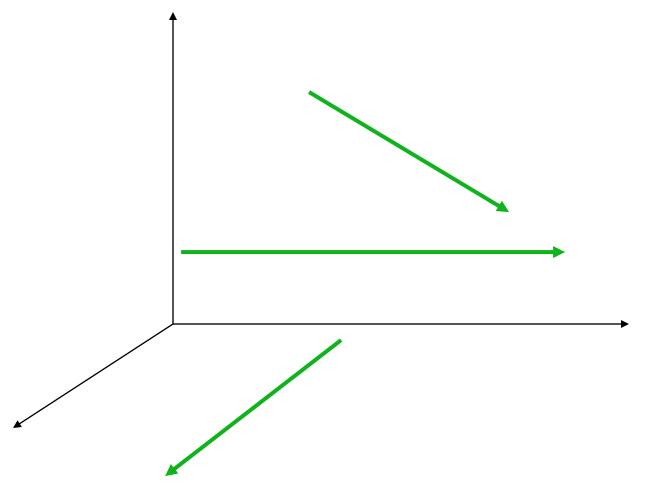
$$GB = \begin{bmatrix} \frac{\sqrt{2\mu C_{OX}}}{C_L} \end{bmatrix} \begin{bmatrix} \sqrt{W} \\ \sqrt{I_{DQ}} \end{bmatrix} \qquad \qquad \text{Increases Quadratically}$$

$$GB = \begin{bmatrix} \frac{2}{V_{DD}C_L} \end{bmatrix} \begin{bmatrix} \frac{P}{V_{EB}} \end{bmatrix} \qquad \qquad \text{Independent of } I_{DQ}$$

$$GB = \frac{1}{\sqrt{I_{DQ}}} \frac{P}{C_L} \sqrt{\frac{2\mu C_{OX}W}{L}} \qquad \qquad \text{Decreases Inverse Quadradicly}$$

$$GB = \frac{\sqrt{\frac{2\mu C_{OX}WP^3}{LV_{DD}}}}{I_{DO}C_L} \qquad \qquad \text{Decreases Inverse Linearly}$$

It depends upon how the design space is explored !!!



Issue becomes more involved for amplifiers or circuits with more than one transistor

Choice of design parameters can have major impact on insight into design

Size of parameter domain should agree with the number of degrees of freedom

Affects of any parameter on performance whether it be in the identified parameter domain or not is strongly dependent on how design space is explored

Small signal and natural parameter domains give little insight into design or performance

Question: How does the A_{V0} of the single-stage amplifier change with V_{EB} ?

$$A_{V0} = \left[\frac{2}{\lambda}\right] \left[\frac{1}{V_{EB}}\right]$$

 A_{V0} decreases with the reciprocal of V_{EB}

Even though there are 2 degrees of freedom, the dependence of V_{EB} on A_{V0} is unambiguous

Question: How does the A_{V0} of the single-stage amplifier change with P?

 A_{V0} is independent of P if V_{EB} is fixed

Even though there are 2 degrees of freedom, the dependence of P on A_{V0} is unambiguous provided V_{FB} is fixed

Question: How does the GB of the single-stage amplifier change with P?

$$GB = \left[\frac{2}{V_{DD}C_{L}}\right] \left[\frac{P}{V_{EB}}\right]$$

GB increases linearly with P?

This is essentially the same question of how GB varies with I_{DQ} Answer depends on how V_{EB} changes

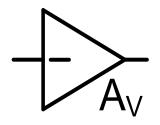
GB increases linearly with P provided the A_{V0} is fixed?

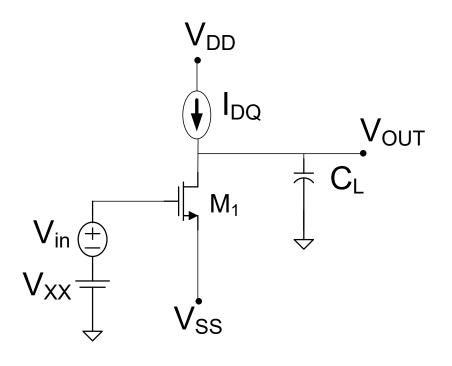
This answer is unambiguous since fixing A_{V0} fixes V_{EB}

Often in situations where the dc gain requirements are fixed and this necessitates a very unfavorable tradeoff between GB and power in this structure since P is a critical "resource" in most applications!

Single-Stage Low-Gain Op Amps

Single-ended input

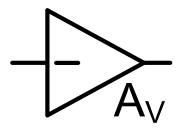




Basic single-stage op amp

Single-Stage Low-Gain Op Amps

Single-ended input



Observations:

- This circuit often known as a common source amplifier
- Gain in the 30dB to 45dB range
- Inherently a transconductance amplifier since output impedance is high
- Voltage gain is ratio of transconductance gain to output conductance
- Critical to know degrees of freedom in design and know how to systematically explore design space
- Alternative parameter domain much more useful for design than smallsignal domain or natural domain
- Performance of differential circuits will be obtained by inspection from those of the single-ended structures

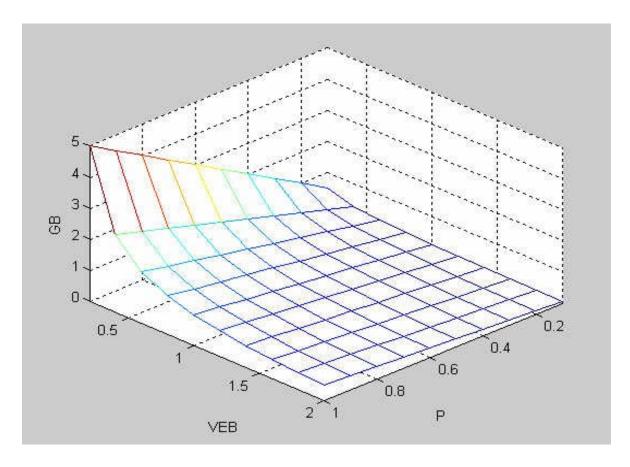
25

Review

- Design space is often a highdimensional system with many local extrema (minimums or maximums)
- Be careful about drawing conclusions about how any parameter individually affects system performance because its affect will depend upon how the design space is explored

Design Space for Single-Stage Op Amp

$$GB = \left[\frac{2}{V_{DD}C_{L}}\right] \left[\frac{P}{V_{EB}}\right]$$



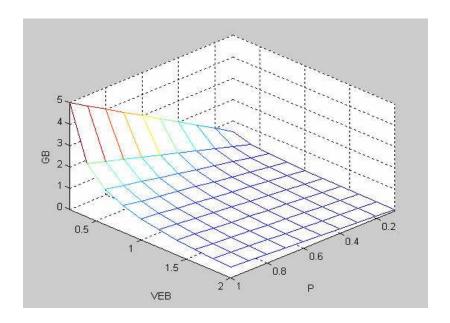
Can we say that GB increases linearly with P?

Review

- Multiple parameter domains can be used to characterize and explore a design space
- Performance characteristics of interest take on many different forms depending upon how design space is characterized
- Critical to identify the real number of degrees of freedom in design space (mathematical degrees of freedom minus the number of constraints)
- Performance characteristics often can be expressed as product of a process dependent term and an architecture dependent term
 - Facilitates comparison of different architectures
- Choice of characterization parameters can make a major difference on how hard it is to explore a design space

Design Space for Single-Stage Op Amp

$$GB = \left[\frac{2}{V_{DD}C_{L}}\right] \left[\frac{P}{V_{EB}}\right]$$



Can we say that GB increases linearly with P?

Can we say that GB increases linearly with P if A_V is fixed?

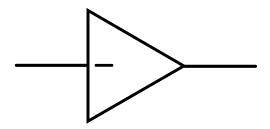
Where we are at: Basic Op Amp Design

- Fundamental Amplifier Design Issues
- Single-Stage Low Gain Op Amps
 - Single-Stage High Gain Op Amps
 - Two-Stage Op Amp
 - Other Basic Gain Enhancement Approaches

Where we are at:

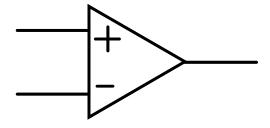
Single-Stage Low-Gain Op Amps

Single-ended input





Differential Input



(Symbol does not distinguish between different amplifier types)

Differential Input Low Gain Op Amps

Will Next Show That:

 Differential input op amps can be readily obtained from single-ended op amps

 Performance characteristics of differential op amps can be directly determined from those of the single-ended counterparts

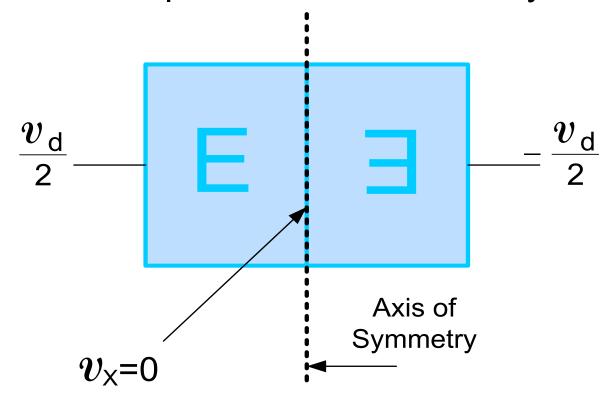
Systematic strategies for designing and analyzing op amps

- Analytical expressions for even simple op amps can become very complicated if brute force analysis techniques are used
- Considerable insight into both performance and design can be obtained from a systematic strategy for design and analysis of op amps
- Most authors present operational amplifiers from an "appear and analyze" approach

A systematic strategy for designing and analyzing op amps will now be developed

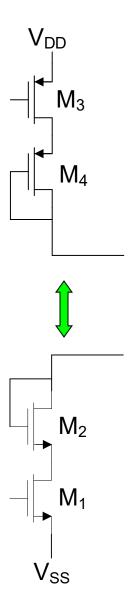
Symmetric Networks

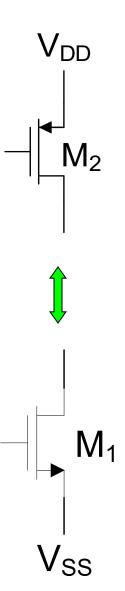
Theorem: If a linear network is symmetric, then for all differential symmetric excitations, the small signal voltage is zero at all points on the axis of symmetry.

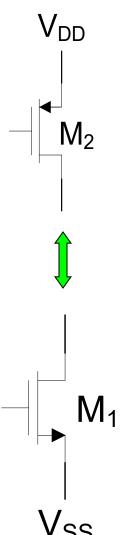


Definition: The counterpart network of a network is obtained by replacing all n-channel devices with p-channel devices, replacing all p-channel devices with n-channel devices, replacing V_{SS} biases with V_{DD} biases, and replacing all V_{DD} biases with V_{SS} biases.

Example:





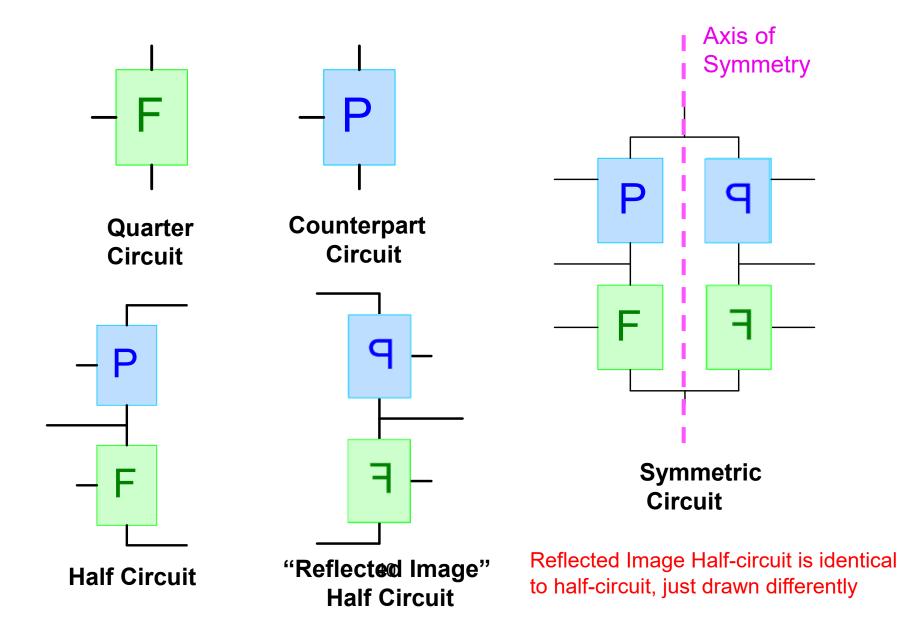


the counterpart network is unique

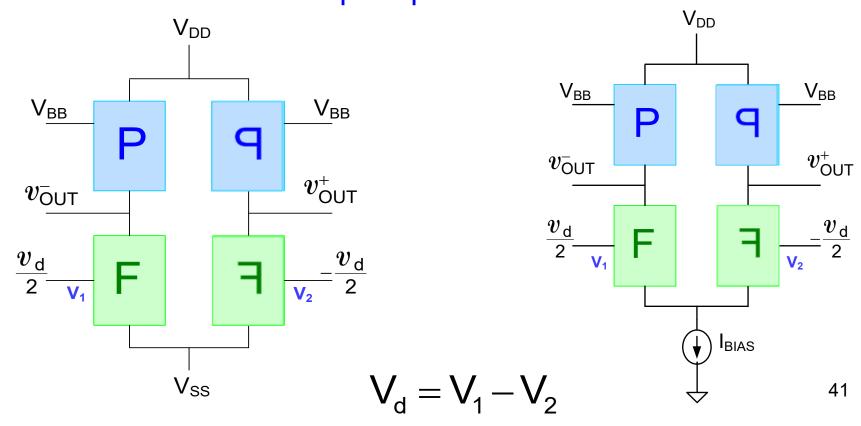
the counterpart of the counterpart is the original network

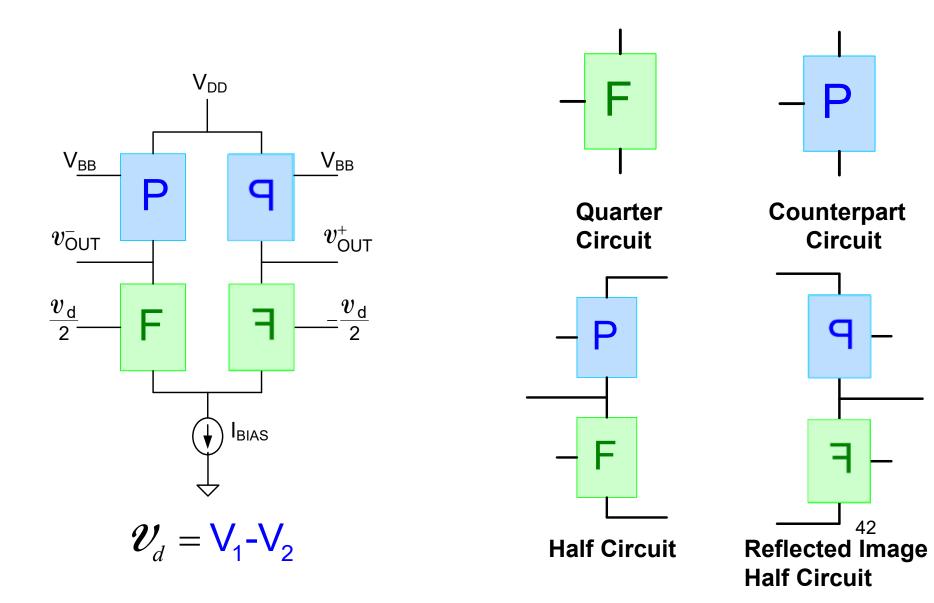
Theorem: The parametric expressions for all small-signal characteristics¹, such as voltage gain, output impedance, and transconductance of a network and its counterpart network are the same.

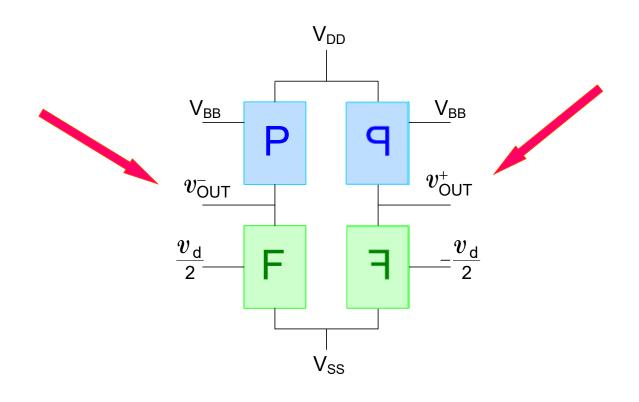
Terminology and Notation



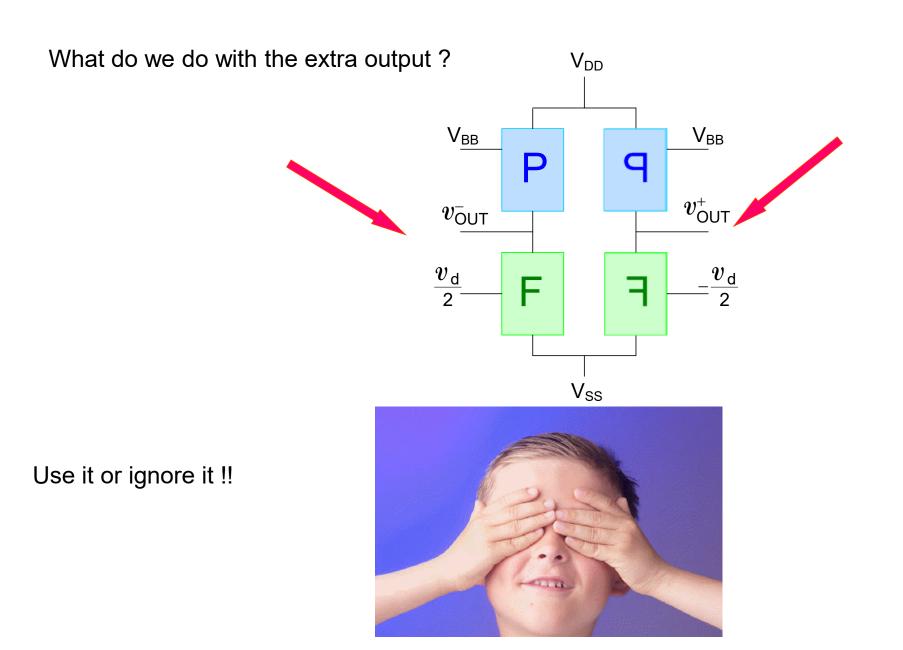
Theorem: If F is any network with a single input and P is its counterpart network, then the following circuits are fully differential circuits --- "op amps".



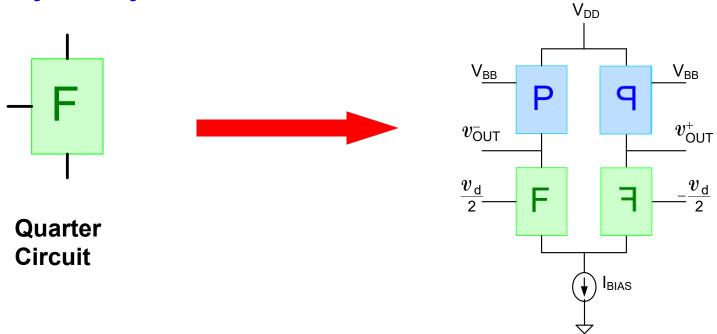




What do we do with the extra output?



A fully differential op amp can be derived from any quarter circuit by combining it with its counterpart to obtain a half-circuit, combining two half-circuits to form a differential symmetric circuit and then biasing the symmetric differential circuit with a current source on the axis of symmetry.



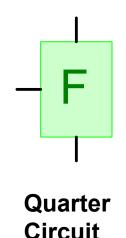
Further, most of the properties of the operational amplifier can be obtained by inspection, from those of the quarter circuit.

45

A fully differential op amp can be derived from any quarter circuit by combining it with its counterpart to obtain a half-circuit, combining two half-circuits to form a differential symmetric circuit and then biasing the symmetric differential circuit with a current source on the axis of symmetry.

Further, most of the properties of the operational amplifier can be obtained by inspection, from those of the quarter circuit.

Implications: Much Op Amp design can be reduced to designing much simpler quarter-circuits where it is much easier to get insight into circuit performance



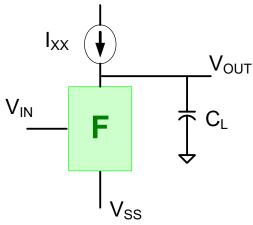
Fully Differential Single-Stage Amplifier



- General Differential Analysis
 - 5T Op Amp from simple quarter circuit
 - Biasing with CMFB circuit
 - Common-mode and differential-mode analysis
 - Common Mode Gain
 - Overall Transfer Characteristics
 - Design of 5T Op Amp
 - Slew Rate

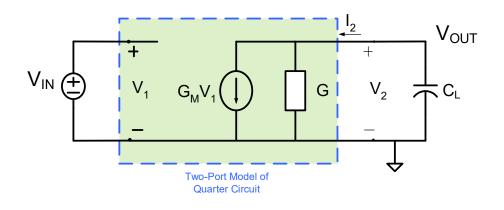
Characterization of Quarter Circuit

If the input impedance is infinite and circuit is unilateral, the two-port network only has two characterizing parameters : G_M and G



$$egin{align*} & egin{align*} & egin{align*$$

$$A_{\text{voqc}} = -\frac{G_{M}}{G}$$

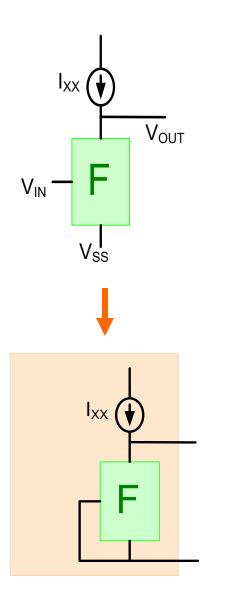


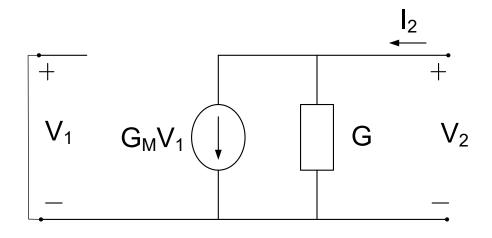
$$\mathbf{A}_{\mathsf{VQC}}(\mathbf{s}) = \frac{-\mathbf{G}_{\mathsf{M}}}{\mathbf{sC}_{\mathsf{L}} + \mathbf{G}}$$

$$BW = \frac{G}{C_1}$$

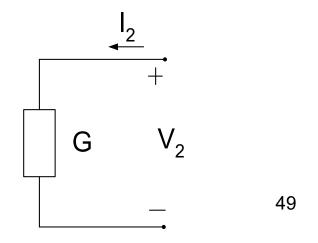
$$\mathbf{GB} = \frac{\mathbf{G}_{\mathsf{M}}}{\mathbf{C}_{\mathsf{L}}}$$

Characterization of Quarter Circuit (or Counterpart Circuit) with input port terminated in small-signal short circuit



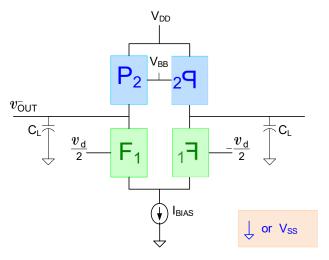


If the input port of a two-port has an ac short, then the two-port reduces to a oneport characterized by the conductance G

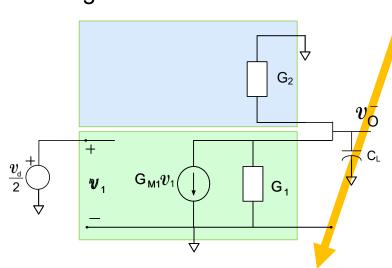


Determination of op amp characteristics from quarter circuit characteristics

-- The "differential" gain --



Small signal differential half-circuit



Derivation:

from KCL and KVL:

$$v_0^-(G_1+G_2+sC_L)+G_{M1}v_1=0$$

$$v_1=\frac{v_d}{2}$$

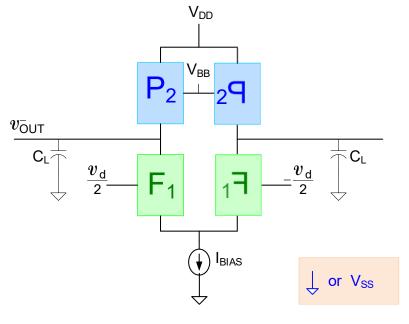
$$A_{V} = \frac{v_{o}^{-}}{v_{d}} = \frac{-\frac{G_{M1}}{2}}{sC_{L} + G_{1} + G_{2}}$$

Note: Factor of 2 reduction of differential gain since only half of the differential input is applied to the half-circuit

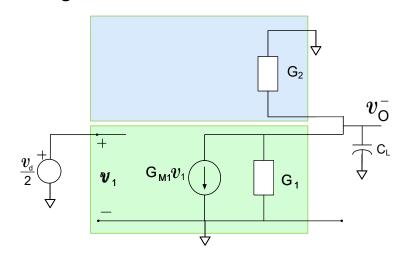
Note: More reduction of gain since denominator increases

 $A_{VQC}(s) = \frac{-G_M}{sC_L + G}$

Determination of op amp characteristics from quarter circuit characteristics -- The "differential" gain --



Small signal differential half-circuit



$$\mathsf{A}_{\mathsf{V}} = rac{v_{\mathsf{o}}^{-}}{v_{\mathsf{d}}} = rac{-rac{\mathsf{G}_{\mathsf{M1}}}{2}}{\mathsf{sC}_{\mathsf{L}} + \mathsf{G}_{\mathsf{1}} + \mathsf{G}_{\mathsf{2}}}$$



BW=? GB=?

$$A_{VO} = \frac{-G_{M1}}{2(G_1 + G_2)}$$

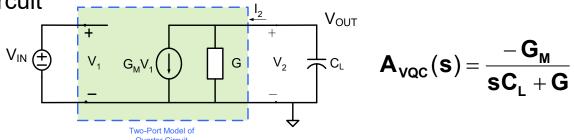
$$BW = \frac{G_1 + G_2}{C_L}$$

$$GB = \frac{G_{M1}}{2C_L}$$

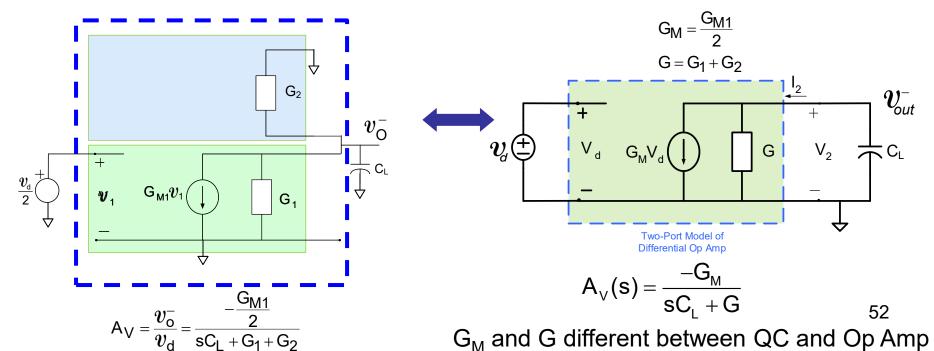
Determination of op amp characteristics from quarter circuit characteristics

-- The "differential" gain --

Small signal Quarter Circuit



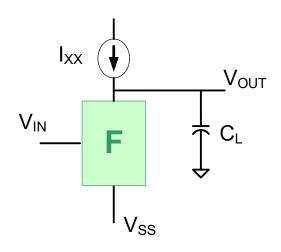
Small signal differential half-circuit (repeated from last slide) and Differential Op Amp



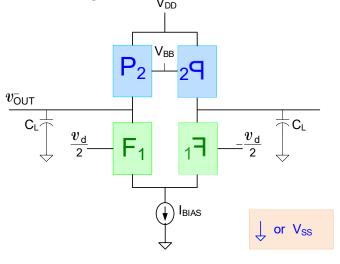
Determination of op amp characteristics from quarter circuit characteristics

-- The "differential" gain --

Small signal Quarter Circuit



Small signal differential amplifier



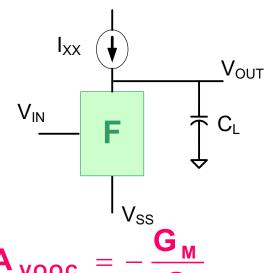
$$\mathbf{A}_{\mathsf{VQC}}(\mathbf{s}) = \frac{-\mathbf{G}_{\mathsf{M}}}{\mathbf{sC}_{\mathsf{L}} + \mathbf{G}} \qquad \qquad \mathbf{A}_{\mathsf{V}} = \frac{v_{\mathsf{O}}^{-}}{v_{\mathsf{d}}} = \frac{-\frac{\mathsf{O}_{\mathsf{M}1}}{2}}{\mathsf{sC}_{\mathsf{L}} + \mathsf{G}_{1} + \mathsf{G}_{2}}$$

Determination of op amp characteristics from quarter circuit characteristics

-- The "differential" gain --

Small signal Quarter Circuit

Small signal differential amplifier



$$A_{\text{voqc}} = -\frac{G_{\text{M}}}{G}$$

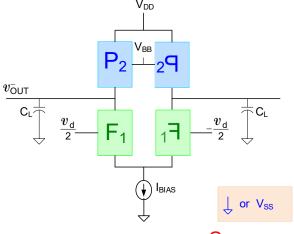
$$BW = \frac{G}{C_L} \qquad GB = \frac{G_M}{C_L}$$

$$W = \frac{G}{C_1} \qquad GB = \frac{G_M}{C_1}$$

Note: Factor of 4 reduction of gain if $G_1=G_2$ (this often occurs)

Note: Factor of 2 increase of BW if $G_1 = G_2$ (this often occurs)

Note: Factor of 2 reduction of GB if $G_1=G_2$ (this often occurs)



$$A_V = \frac{v_o^-}{v_d} = \frac{-\frac{G_{M1}}{2}}{sC_L + G_1 + G_2}$$

$$A_{V0} = \frac{v_{OUT}^{-}}{v_{d}} = \frac{-G_{M1}}{2(G_{1} + G_{2})}$$

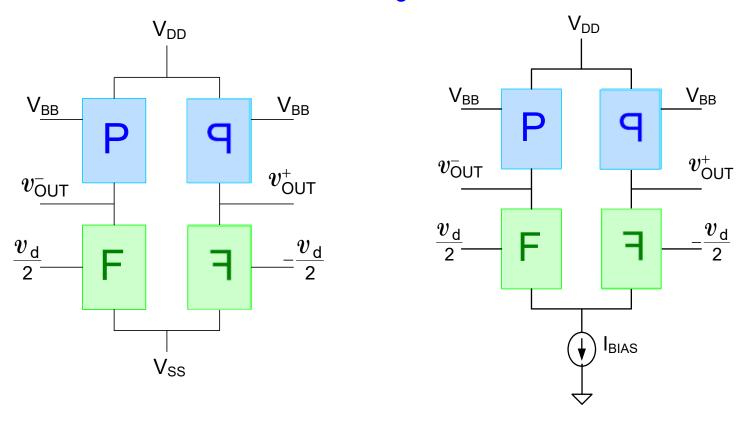
$$BW = \frac{G_1 + G_2}{C_1}$$

$$GB = \frac{G_{M1}}{2C_{I}}$$

54

Comparison of Tail Voltage and Tail Current Source Structures

-- The "differential" gain --



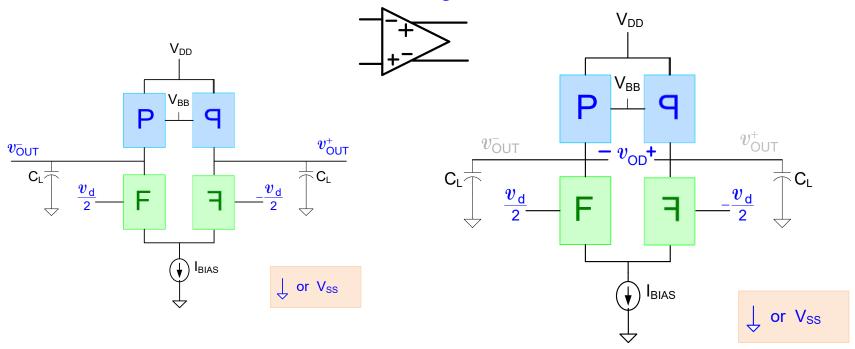
Small signal half-circuits are identical so differential voltage gains, BW, and GB are all the same

Biasing Issues for Differential Amplifier

 Tail voltage bias not suitable for large common-mode (CM) input range but does offer good output swing

 Tail current bias provides good CM input range but at the expense of a modest reduction in output signal swing

Differential Output Amplifiers -- The "differential" gain --



Single-Ended Outputs

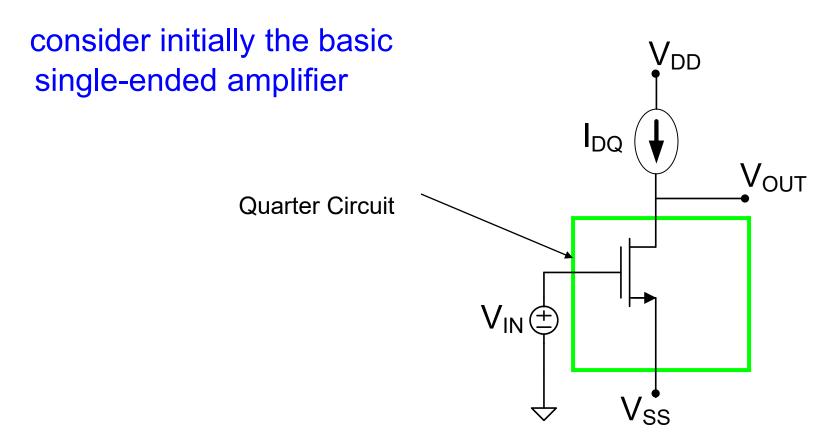
Differential Output

Theorem: For a symmetric circuit with symmetric outputs and differential excitations:

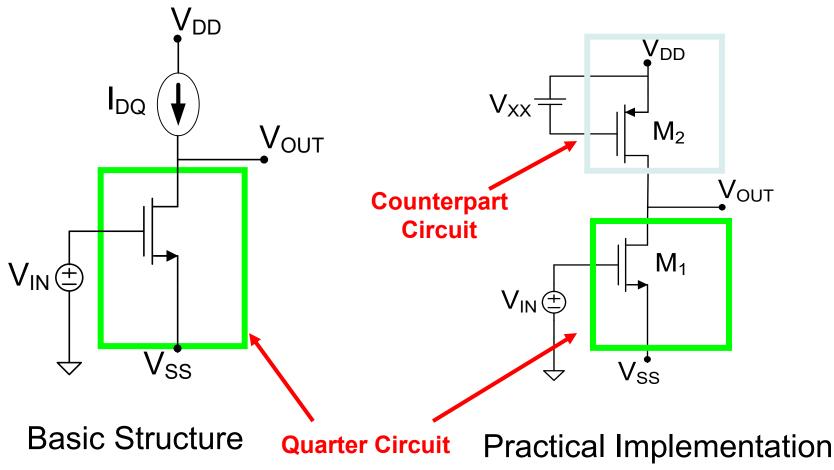
- Differential Voltage Gain Double that of Single-Ended Structure
- BW is the same
- GB Doubles for the Differential Output Structure

- Fully Differential Single-Stage Amplifier
 - General Differential Analysis
- 5T Op Amp from simple quarter circuit
 - Biasing with CMFB circuit
 - Common-mode and differential-mode analysis
 - Common Mode Gain
 - Overall Transfer Characteristics
 - Design of 5T Op Amp
 - Slew Rate

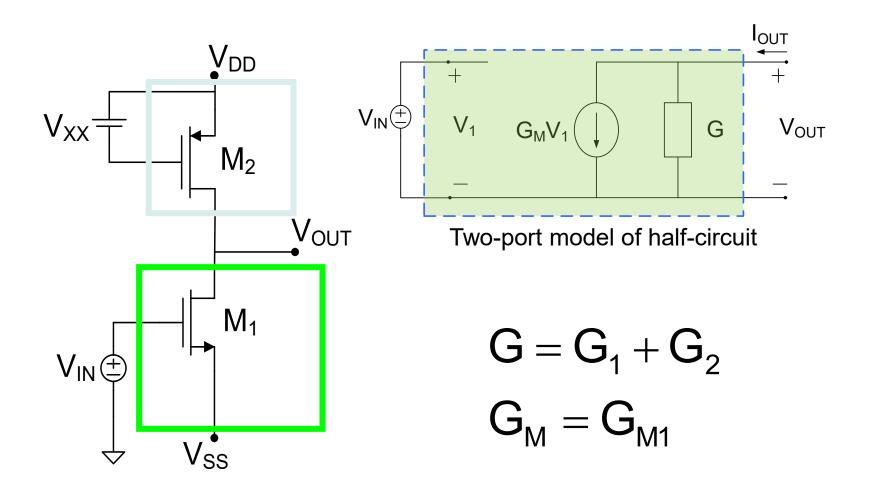
Applications of Quarter-Circuit Concept to Op Amp Design



Single-stage single-input lowgain op amp



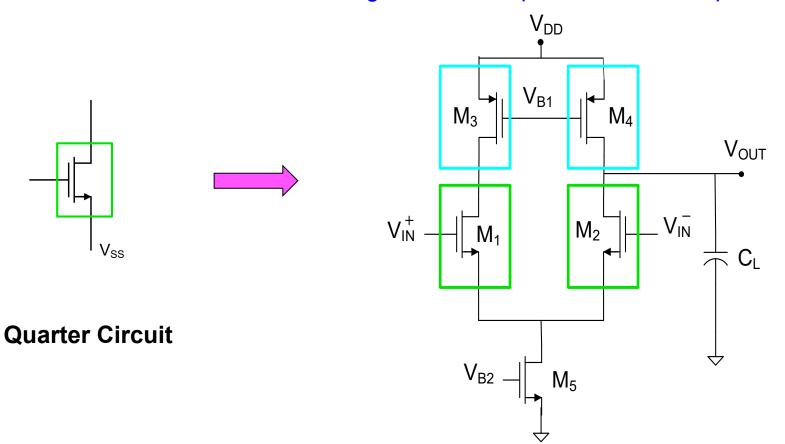
Small signal model of half-circuit



Single-stage low-gain differential op amp

-- The "differential" gain --

Single-Ended Output : Differential Input Gain



Have synthesized fully differential op amp from quarter circuit!

Termed the 5T Op Amp

Will determine small-signal properties of 5T op amp by inspection (next time)



Stay Safe and Stay Healthy!

End of Lecture 3